Measure and Proportion in the Monumental Gabled Altarpieces of Duccio, Cimabue, and Giotto

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Recent investigations into the complex problem of design and proportion in Italian art have revealed new evidence for the possible application of geometric and harmonic systems in the design of late mediaeval carpentry and early Renaissance sculpture. Working from the point of view of solving problems of reconstruction, John White has found that the carpentry proportions of Duccio's dismembered *Maestà* as well as the measurements of Donatello's Santo Altar at Padua reflect a geometrical system based on the square.[1] Diane Finiello Zervas has discovered by analysing the *St. Matthew* tabernacle at Orsanmichele that Ghiberti employed a system of design based on the harmonic or musical proportions.[2] Whereas White has isolated incommensurable ratios among his measurements which reflect geometrical processes, Zervas has found simple, commensurable ratios in the proportions of Ghiberti's tabernacle which coincide with the Pythagorean-Platonic musical scale.

Although such important studies are based on an extensive analysis of the actual measurements of the monuments and contribute much valuable information to our understanding of the practical use of proportional rules in design in the fourteenth and fifteenth centuries, they tend to concentrate on the data obtained from a single, isolated work of art rather than a comparative analysis of a group of related monuments. The present study attempts to discover whether any positive results can be derived from the examination of a series of incontestably important Italian altarpieces of a similar structural and iconographical nature produced within a closely delimited span of time. An analysis of the carpentry proportions of the monumental gabled altarpieces of Duccio, Cimabue, and Giotto, designed and painted between ca. 1280 and ca. 1310, discloses evidence of a similar pattern of proportional relationships and design processes. This pattern is geometrical in nature and is derived fundamentally from the *sides* and *diagonals* of squares and the generation of root rectangles. The evidence of incommensurable ratios in the carpentry proportions of the complete retables from the late duecento and early trecento in Tuscany tends to support the observations of White with regard to the design and proportions of Duccio's contemporaneous yet dismantled *Maestà*.

The design principle based on the side and diagonal of the square and the generation of root rectangles can be illustrated in the structure of the front predella of Duccio's *Maestà*.[3] The front predella is the best preserved part of the dismembered carpentry of Duccio's altarpiece, and its structure provides the clearest evidence of a consistent proportional arrangement. Originally the predella fields and frames were contained on a single plank of wood; the frames were attached to the groundboard and the layers of gesso, gold leaf, and paint were applied in succession once the framing elements were in place. Duccio composed the *Infancy of Christ* in seven narrative

[1] J. White, "Donatello's High Altar in the Santo at Padua," *Art Bulletin*, LI (1969), 1–14, 119–41; and "Measurement, Design and Carpentry in Duccio's *Maestà*," *Art Bulletin*, LV (1973), 334–66, 547–69.

I am grateful to Joan Brink for having converted my measurements into the series of scale drawings reproduced in this article.

[2] D. Finiello Zervas, "Ghiberti's St. Matthew Ensemble at Orsanmichele: Symbolism in Proportion," *Art Bulletin*, LVIII (1976), 36–44.

[3] White's discussion of the front predella in "Measurement, Design and Carpentry," pt. 1, contains only a partial analysis of its actual proportional structure. A review of this by the author is in progress.

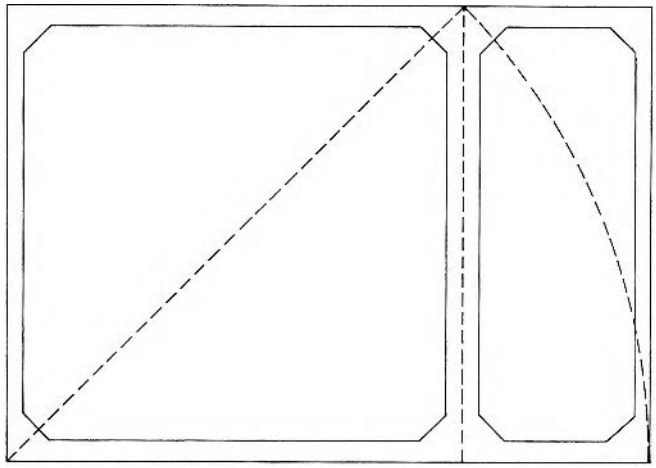

DIAGRAM A. Duccio di Buoninsegna, *Maestà*, Front Predella, Composite *Infancy* and *Prophet* Panel with Geometrical Schemata.

fields, with six oblong fields placed in between to represent the Old Testament *Prophets*. The pictorial composition was intended to emphasize the reciprocal relations between the Old Testament prophecies and their fulfilment in the early life of Christ. These harmonious relationships in content also apply to the geometrical structure of the front predella. White's measurements indicate conclusively that Duccio designed his narrative and oblong fields according to a specific rule of proportion governed by the incommensurable relationship between the side of a square and its diagonal, or the ratio of $1:\sqrt{2}$. Each of the seven *Infancy* panels plus their immediate frames were designed as squares, and the width of the *Prophet* panels was established by the diagonal of these square narrative fields (Diagram A). The composite *Infancy* and *Prophet* panels are therefore each root-two rectangles because their height is related to their width as $1:\sqrt{2}$. Duccio also uses this incommensurable proportion for the internal field of his *Madonna of the Franciscans* in the Siena Pinacoteca, and it can also be observed in the proportions of the compartmental altarpieces produced in his workshop.[4] Furthermore, Simone Martini, one of Duccio's disciples, took a particular delight in this proportion since a number of his panel paintings reflect this structure.[5]

In addition to employing the root-two rectangle in the design of their panel paintings, Duccio and his contemporaries appear to have understood how to use a complete series of measures governed by the same incommensurable ratio of $1:\sqrt{2}$. White has illustrated this in the carpentry proportions of the *Maestà*, and it is a phenomenon also reflected in the proportional structure of Simone Martini's altarpieces.[6] The generation of such geometrical progressions can be obtained from the *sides* of successively inscribed squares. The principle of the inscribed square is described as the "Theorem of Plato" in the introduction to the ninth book of Vitruvius's *De Architectura*, and it occurs as well in the additions of Magister 2 to the Sketchbook of Villard d'Honnecourt. The inscribed quadratic figure was published in the fifteenth century by the German master-mason Matthias Roriczer, who illustrated its practical application to the design of a simple finial.[7] Frankl has described the geometrical configuration as the "secret" of the mediaeval masons,[8] and the principle appears to have been employed quite extensively in mediaeval and Renaissance design.

Measuring the *sides* of inscribed squares produces a series of linear values characterized by the ratio of $1:\sqrt{2}:2:2\sqrt{2}$ etc. (Diagram B). White has discussed the mathematical characteristics of this progression in his study of the *Maestà*, but its most important

[4] The incommensurable ratio of $1:\sqrt{2}$ occurs throughout the carpentry proportions of polyptychs 28 and 47 in the Siena Pinacoteca, both attributed to Duccio and his workshop.

[5] Simone Martini's *Christ's Return from the Temple* in Liverpool is a root-two rectangle, as are the complete panels of the dismembered Orsini Polyptych in Antwerp and Paris, and the *Virgil Frontispiece* in the Biblioteca Ambrosiana in Milan. See my "Simone Martini's 'Orsini Polyptych,'" *Jaarboek Van Het Koninklijk Museum Voor Schone Kunsten–Antwerpen*, 1976, 7–23. For Vitruvius's description of the root-two rectangle as a suitable shape for a cella see *De Architectura*, vi, iii, 3.

[6] A survey of the carpentry proportions of the major altarpieces from the late duecento and early trecento in Tuscany reveals that the proportional pattern which White discovered in the carpentry dimensions of the *Maestà* is not an isolated case, but is found as well in the panel paintings of his followers.

[7] Matthias Roriczer, *Puechlein der Fialen Gerechtigkeit* (Regensburg, 1486), facs. ed. K. Schottenloher (Regensburg, 1923).

[8] P. Frankl, "The Secret of the Medieval Masons," *Art Bulletin* XXVII, (1945), 46–60.

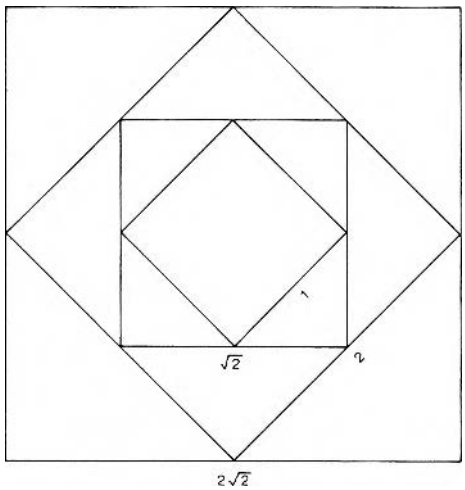

DIAGRAM B. Inscribed Square Configuration with Geometrical Ratios.

qualities for our investigation are twofold: (1) each adjacent value is incommensurable as the side of a square is to its diagonal, and (2) the alternate stages of the progression are commensurable, measuring double or half one another — hence the ratio $1:\sqrt{2}:2$.

Once the geometrical construction of the root-two rectangle is set out and the mathematical characteristics of the inscribed square principle are established, it is possible to examine the gabled altarpieces and to illustrate in a comparative analysis their proportions and design. Owing to the generous assistance of the officials of the Uffizi and the Louvre, it was possible to study in detail the carpentry of four of the principal gabled retables from the late Middle Ages in Tuscany and to record their exact dimensions.[9] These panels include the three monumental altarpieces in the first room of the Uffizi: Duccio's Rucellai altarpiece from Santa Maria Novella (Fig. 1), Cimabue's altarpiece from Santa Trinita (Fig. 2), and Giotto's Ognissanti altarpiece (Fig. 3); and the large retable in the Louvre attributed to Cimabue and his workshop and originally designed for San Francesco in Pisa (Fig. 4). These panel paintings together form an homogeneous group of altarpieces produced in Florence and Pisa between ca. 1280 and ca. 1310 by the major painters of the period. Except for Cimabue's altarpiece for Santa Trinita, which has had the tip of the gable restored and the frame replaced, the carpentry of each of the retables is in excellent physical condition with the original proportions of the fields and frames preserved intact.

A comparative analysis of the width dimensions of the four gabled altarpieces from Florence and Pisa reveals a similar proportional structure in their fields and framing elements. The width of the continuous outer frame is in each case derived from a geometrical progression based on either the internal width or the total width of the altarpiece. The given width is reduced geometrically to a series of linear values having the same mathematical characteristics as the proportional sequence described above ($1:\sqrt{2}:2:2\sqrt{2}$ etc; Table 1). For example, the internal field width of Duccio's Rucellai altarpiece is 233.4 cm and the frame measures 29.2 cm in width.[10] If the inscribed

[9] The dimensions of the altarpieces were recorded with a tape measure, and the greatest care was exercised to obtain accurate results. At the Uffizi a mechanical lift was employed to reach the highest points and the gallery officials assisted in taking the measurements. At the Louvre our examination was assisted by one of the conservators, Mlle. Claudie Ressort, and in all of our investigations special consideration was given to the carpentry construction including later restorations to the wooden format.

[10] The width dimensions of Duccio's Rucellai altarpiece will also translate into the Florentine *braccio* system: the frame is one-half *braccio* wide and the internal width is equal to three *bracci*. This system of measurement is not reflected as clearly in the panel's height dimensions nor in the proportions of the other gabled altarpieces under consideration.

	Duccio Rucellai	*Cimabue Louvre*	*Giotto Ognissanti*	*Cimabue S. Trinita*
1	**233.4 cm**	**239.5 cm**	**203.2 cm**	**242.0 cm**
2	165.0	169.3	143.6	171.1
3	116.7	119.7	101.6	121.0
4	82.5	84.6	71.8	85.5
5	58.3	59.8	50.8	60.5
6	41.2	42.3	35.9	42.7
7	**29.2** (29.2)	29.9	25.4	30.2
8		**21.2** (20.0)	18.0	21.3
9			**12.7** (12.7)	15.6
10				**10.6** (10.6)

TABLE 1. Derivation of the Frame Dimension from a Geometrical Progression based on the Given Width of the Altarpiece (given width and ideal frame dimension in bold type, actual frame dimension in parentheses).

FIGURE 1. Duccio di Buoninsegna, *Rucellai Altarpiece*, 1285. Florence, Uffizi (Photo: Gabinetto Fotografico – Sopr. Gallerie).

FIGURE 2. Cimabue, *Santa Trinita Altarpiece*, early 1280s. Florence, Uffizi (Photo: Gabinetto Fotografico – Sopr. Gallerie).

square progression is begun with the internal width, then the dimension of the framing element is the seventh value in the sequence: that is, the side of the seventh consecutive square within the geometrical configuration. This pattern also applies to the width dimensions of Cimabue's panel in the Louvre as well as Giotto's Ognissanti altarpiece. In the first, the internal width (239.5 cm) produces the width of the frame as the eighth digit in the progression; in the second, the internal width (203.2 cm) generates the width of the frame (12.7 cm) as the ninth value in the sequence. Of the four altarpieces, only Cimabue's for Santa Trinita derives its framing dimension from the total width of the retable rather than the internal field width. The altarpiece measures 242.0 cm and the restored frame (which duplicates the width of the original element[11]) is 10.6 cm or the tenth digit in the geometrical progression.

Each of the three artists, therefore, selects the measure of the framing element from a series of proportionate lengths based on the given width of the altarpiece. Significantly, the width of the frame in the four retables examined *never* coincided with the same digit in the geometrical progressions but varied from the seventh, eighth, ninth, and tenth consecutive values. This phenomenon illustrates conclusively that

[11] Since the restored frame of Cimabue's Santa Trinita altarpiece extends from the original gesso ground to the edge of the backboard (which has not been cut), it must duplicate the width of the original framing element. Similarly, the restored tip of the gable approximates its original slope thus making the height of the altarpiece reasonably accurate.

FIGURE 3. Giotto, *Ognissanti Altarpiece*, ca. 1310. Florence, Uffizi (Photo: Gabinetto Fotografico – Sopr. Gallerie).

FIGURE 4. Cimabue, *San Francesco Altarpiece*, early 1290s. Paris, Louvre (Photo: Cliché des Musées Nationaux).

even though the frame in each of the four retables is in a slightly different proportional relationship with the given width, they are all nevertheless generated consistently from the same geometrical principle based on the square.

The geometry of the square and the generation of root-rectangles from the diagonal are moreover instrumental in obtaining the height dimensions of each of the four gabled altarpieces. The external shoulder height of Duccio's Rucellai altarpiece, for example, is produced from the measure of the diagonal of a root-two rectangle erected on the total width of the panel (Diagram C). The root-three diagonal measures 357.3 cm,[12] and this contrasts closely with the actual height of the shoulder at 356.0 cm. Cimabue employs exactly the same geometrical process to obtain the total height of his crucifix for Santa Croce, and a similar process is evident in the proportions of his retable for Santa Trinita (Diagram D). Cimabue constructs a root-two rectangle on the total width of the altarpiece (242.0 cm) and the approximate height of the internal shoulder is derived from the 342.2-cm height measure. Cimabue then employs the root-three diagonal (419.1 cm) as the approximate height of the altarpiece (423.3 cm). This geometrical process produces a root-three rectangular shape for the

[12] The ideal values for the diagonals are computed by multiplying the given measures by the numerical equivalent of root-two (1.414) or root-three (1.732). In the case of the shoulder height of the Rucellai panel, the root-three diagonal of 357.3 cm is equal to 206.3 cm multiplied by 1.732.

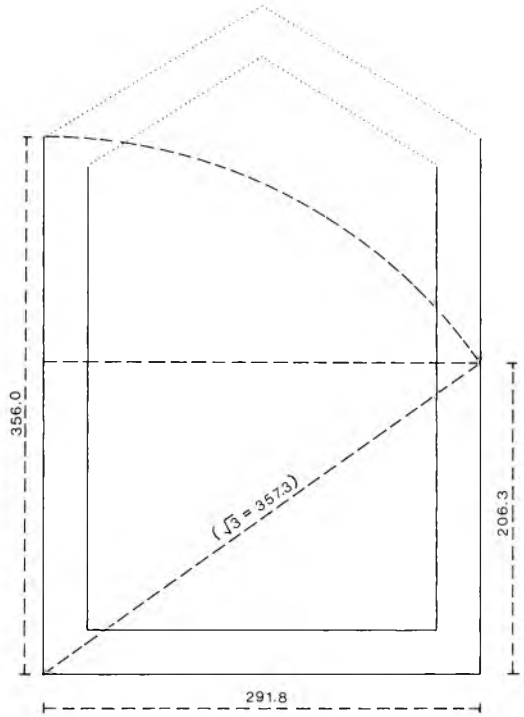

DIAGRAM C. Duccio di Buoninsegna, *Rucellai Altarpiece*. Florence, Uffizi. (Diagrams C–J, scale 1:50.)

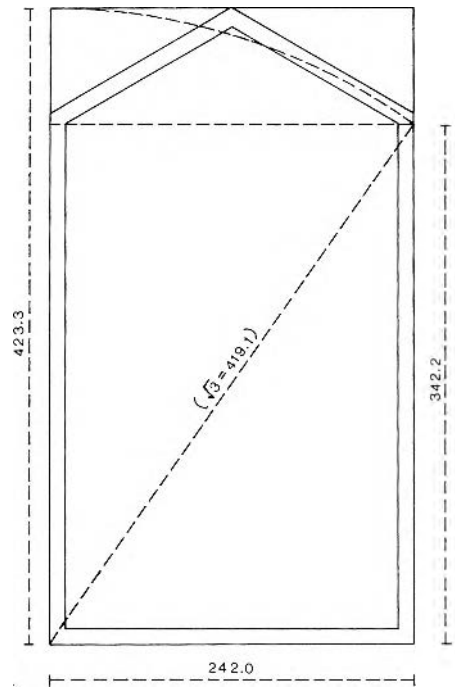

DIAGRAM D. Cimabue, *Santa Trinita Altarpiece*. Florence, Uffizi.

retable, indeed a suitable field shape for an altarpiece designed for the Church of the *Trinity*.

The external shoulder height of Cimabue's panel in the Louvre is also based on the geometry of the square (Diagram E). Here Cimabue squares the internal width (239.5 cm) to produce a root-two diagonal of 338.6 cm and this measure provides the shoulder heights of 337.1 cm and 338.5 cm for the two sides of the altarpiece. Comparing this procedure with the design of Cimabue's crucifix for Santa Croce illustrates some interesting concordances (Diagram F). The height of Christ (including his nimbus) from the base of the crucifix as well as the height of the terminal compartments containing the images of the Virgin and St. John the Evangelist measure 336.0–338.0 cm, or the shoulder height of Cimabue's Louvre retable. Moreover, the total reconstructed height of the Santa Croce crucifix is 478.0 cm or the diagonal of a 337.9-cm square. It would appear, therefore, that Cimabue and his workshop were employing the same sequence of measures in the two monumental panel paintings. This is by no means an isolated case. Simone Martini, for example, often applies the same family of related progressions in the proportions of his carpentry designs and Ambrogio Lorenzetti's altarpieces also reflect similar phenomena.

Not only is this evidence valuable in reconstructing the dimensions of lost parts of dismembered panels, but it also provides potential assistance when used in conjunction with stylistic and other physical evidence in solving problems of attribution. The carpentry proportions of Giotto's Ognissanti altarpiece relative to the carpentry proportions of his often attributed crucifix in Santa Maria Novella (Fig. 5) provide an interesting model. The external shoulder height of the Ognissanti panel (Diagram G) is obtained from the measure of the diagonal of a square erected on the internal width, which is precisely the same process that Cimabue uses in the Louvre retable for the corresponding element. The root-two diagonal measures 287.3 cm and this coincides with the shoulder height at 285.0–287.0 cm. If we then examine the proportions of the Santa Maria Novella crucifix, we discover that the designer of the cross employs measures from the same geometrical progression that is used in the Ognissanti altarpiece. According to the data given in the catalogue of the *Mostra Giottesca* of 1937,[13] the crucifix measures 578.0 cm high by 406.0 cm wide. If these dimensions are correct, they

[13] G. Sinabaldi and G. Brunetti, *Pittura Italiana del Duecento e Trecento: Catalogo della mostra Giottesca di Firenze del 1937* (Florence, 1937), 301.

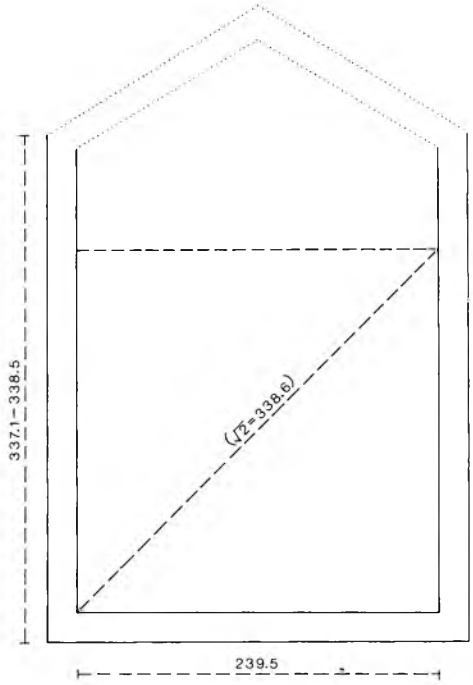

DIAGRAM E. Cimabue, *San Francesco Altarpiece*. Paris, Louvre.

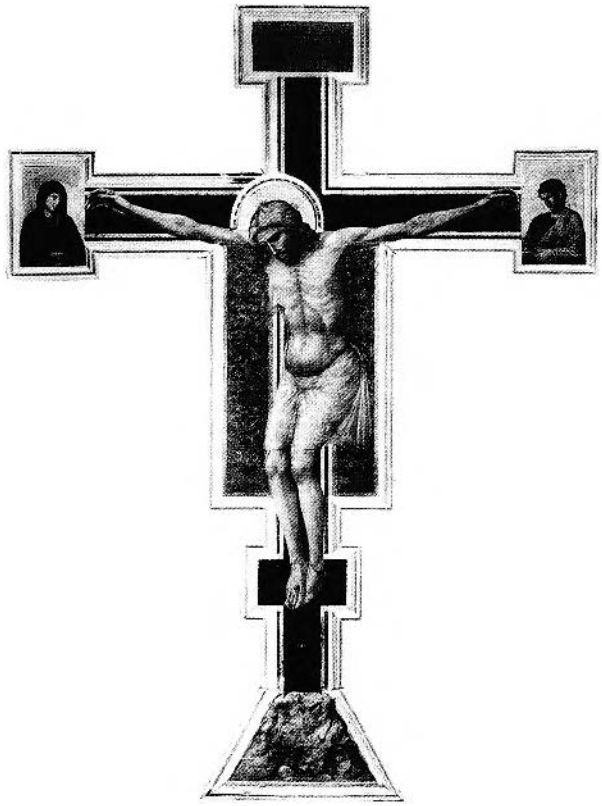

FIGURE 5. Giotto (attrib.), *Santa Maria Novella Crucifix*, 1290s. Florence, Santa Maria Novella, Sacristy (Photo: Gabinetto Fotografico – Sopr. Gallerie).

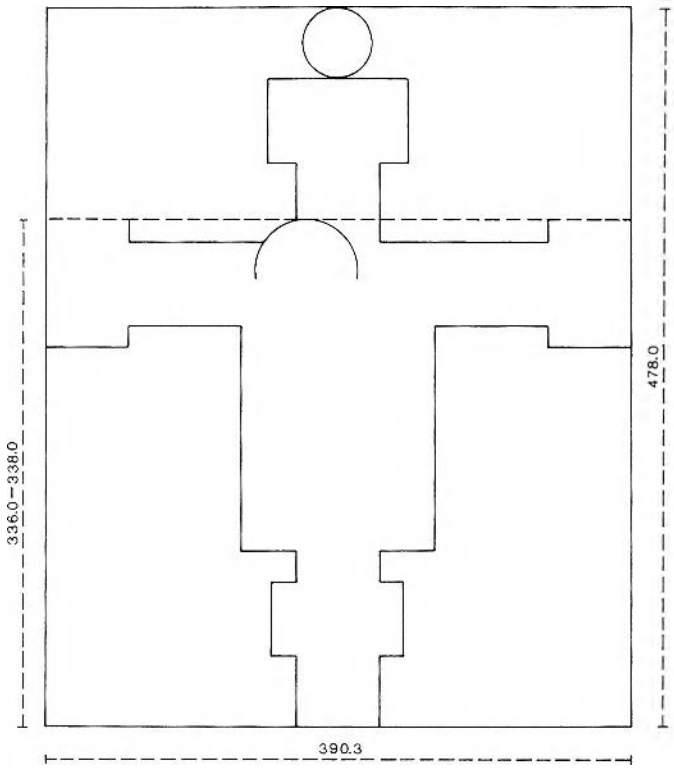

DIAGRAM F. Cimabue, *Santa Croce Crucifix*. Florence, S. Croce, Refectory.

indicate that the height of the crucifix is only 3.4 cm more than the shoulder height of Giotto's altarpiece doubled, and the width of the crucifix is a mere 4 mm in excess of the internal width of the altarpiece doubled.[14] According to the given dimensions, therefore, the monumental crucifix describes a root-two rectangle based on the same sequence of geometrically derived measures as were used in the design of Giotto's Ognissanti altarpiece. These proportional correlations suggest that the two panel paintings may have been designed with the same carpenter's square, and this possibility contributes new support to the argument that, like the altarpiece, the crucifix undoubtedly comes out of Giotto's workshop and is most likely by the master himself.

To complete the height proportions of the Ognissanti, Rucellai, and Louvre altarpieces, Giotto, Duccio, and Cimabue each square the total width of the panel, then take the measure of the diagonal as the key height dimension. The square diagonal of Giotto's Ognissanti retable (Diagram H) is 325.2 cm and this dimension coincides almost exactly with the internal height at 325.0 cm. Duccio and Cimabue follow nearly the same process: the root-two diagonal of the Rucellai altarpiece (Diagram I) measures 412.6 cm and this corresponds closely with the height of 409.4 cm from the base of the panel to the inside of the gable; and the root-two diagonal of the Louvre panel measures 395.2 cm, thus providing the dimension of 398.3 cm for the corresponding height (Diagram J).

Once the measure and proportions of the carpentered superstructures are known, one is led to examine the pictorial compositions themselves. Had the artist sought to design the carpentry according to specific geometrical rules based on the square (which would ensure a proportional structure in the shape, field, and frame of his altarpiece), it seems possible that he would also have attempted to set out the principal lines and areas of his pictorial composition in accordance with the same design principles. Even though the geometrical schema of the carpentry design may in some instances coincide with elements within the pictorial field, it is not possible at this time to indicate a comprehensive pattern of relationships. It *is* possible, however, to illustrate in three of the gabled altarpieces an approximate concordance between the carpentry design and the positioning of an important gesture within the pictorial field.

Perhaps the most significant gesture in the pictorial compositions — and one whose placement the painter would surely have calculated with some degree of precision — is the blessing hand of Christ. In the

[14] Since the crucifix is situated high upon the sacristy wall of Santa Maria Novella, it is virtually impossible to examine its superstructure closely. It is feasible that a roundel containing a *Christ the Redeemer* originally surmounted the cross, though confirmation of this must wait until a complete survey of its structure and proportions can be undertaken.

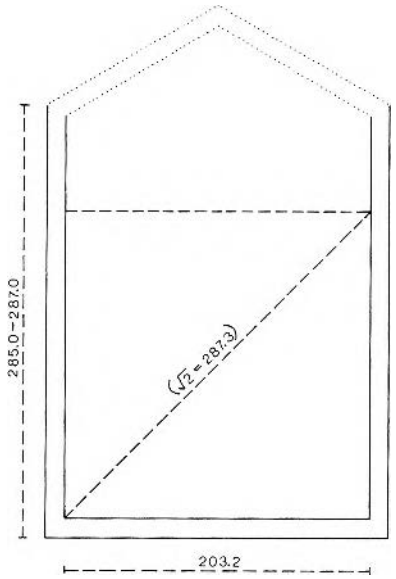

DIAGRAM G. Giotto, *Ognissanti Altarpiece*. Florence, Uffizi.

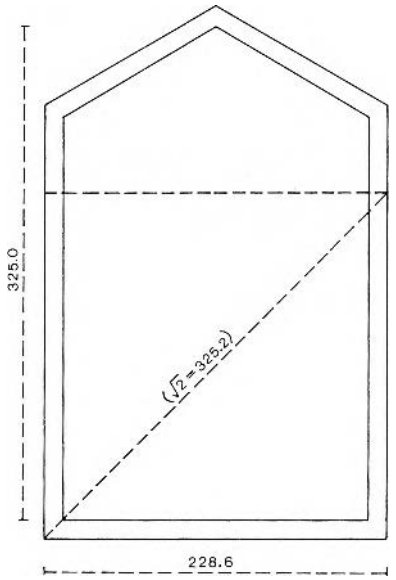

DIAGRAM H. Giotto, *Ognissanti Altarpiece*. Florence, Uffizi.

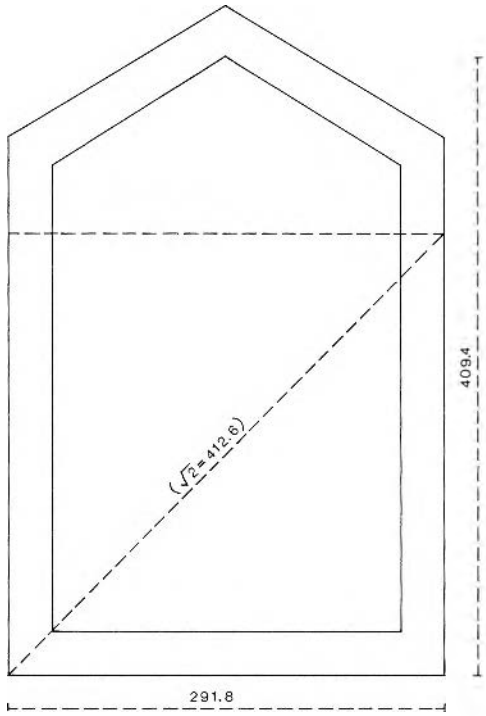

DIAGRAM I. Duccio di Buoninsegna, *Rucellai Altarpiece*. Florence, Uffizi.

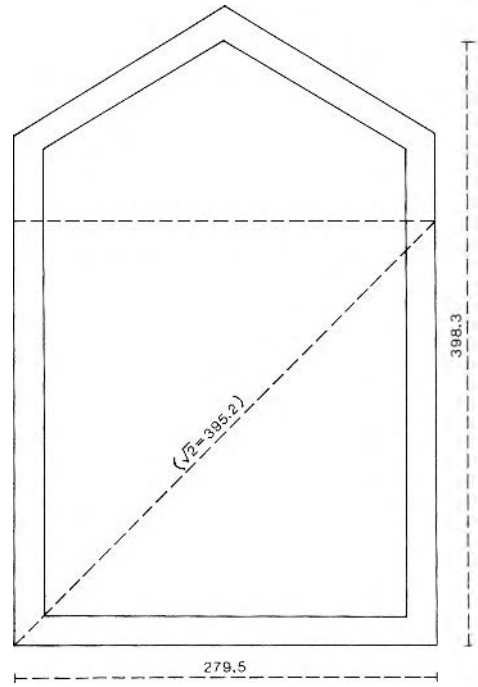

DIAGRAM J. Cimabue, *San Francesco Altarpiece*. Paris, Louvre.

Rucellai, Louvre, and Ognissanti altarpieces, the hand of Christ is situated in what appears to be an identical position inside the field. This is near the central vertical axis and at a height from the top of the base frame equal to the internal width of the altarpiece. In other words, a square erected on the base of the pictorial field equal to its breadth provides the position of the blessing hand of Christ. Although this observation must necessarily remain somewhat tentative, because it is based on the evidence of photographs rather than accurate measurements, it does correlate consistently with the principles and processes active in the design of the carpentry, and it provides a clue to the probable unity of carpentry and pictorial design in the monumental gabled altarpieces of Duccio, Cimabue, and Giotto.

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